EARTHQUAKES AND GEOMBINATORICS

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Abstract. According to modern geophysics, earthquakes mainly occur in the places where tectonic plates interact.

Plate tectonics started with analyzing the simplest plate interactions: *heads on* collisions and *pull apart* motions; such interactions are the most common. Corresponding interaction zones are often very seismically active; in addition to frequent small and medium earthquakes, they host the most destructive earthquakes.

Not so common are *oblique* plate collisions (in which plates collide at an oblique angle) and lateral motions. Earthquakes caused by these non-canonical interactions are not so destructive, but they can be even more dangerous that heads-on ones, because in heads-on collisions, small earthquakes serve as a warning for a big one, while in oblique and lateral collisions zones, earthquakes can occur without warning.

To predict such un-warned earthquakes better, it is necessary to look for traces of such quakes in the geological past. This necessity raises an important question: currently, non-canonical collision zones are relatively rare. Are we sure that they occured in the past at all? Maybe, our search for such past quakes is futile?

To answer this question, in the present paper, we reformulate it in topological and geometric terms. Our answer: it is impossible to have only heads-on and pull-apart collisions, so there must have been oblique and/or lateral collisions in each past geological epoch.

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1. EARTHQUAKES AND PLATE TECTONICS: THE BASIC CONNECTION AND THE FORMULATION OF OUR PROBLEM

According to modern geophysics (see, e.g., (McKenzie Parker 1967), (Cox 1973), (Plate 1980), (Cox 1986), and (Condie 1989)), the surface of the Earth is divided into several *tectonic plates* that move relative to each other, and earthquakes mainly occur in the zones where two plates interact.

Plate tectonics started with analyzing the simplest plate interactions: *heads on* collisions and *pull apart* movements, in which both plate move in the direction orthogonal to their common boundary. Such interactions are most common. Corresponding interaction zones are often very seismiccally active; in addition to frequent small and medium earthquakes, they host the most destructive earthquakes.

When plates collide in such "canonical" manner, they inevitably crash into each other, and earthquakes occur. In such zones, it may be difficult to predict when exactly a large earthquake will occur, but the very seismicity of this zone is clear.

Not so common are *oblique* plate collisions (in which plates collide at an oblique angle) and lateral motions. Their seismicity is different from seismicity in canonical collision zones:

- On one hand, earthquakes caused by these non-canonical interactions are usually not so destructive as earthquakes in heads on collision zones.
- On the other hand, earthquakes in non-canonical collision zones are much more difficult to predict than earthquakes in head on collision zones: Indeed, a more complicated character of the plate motion creates several different faults with different motions/orientations, and we can't as easily tell which faults are dangerous.

In view of this "more unpredictable" character of earthquakes in non-canonical zones, it is necessary to analyze earthquakes in such zones. Since these zones are not so common, to get a good statistical analysis, we must analyze not only current earthquakes in such zones, but we must also take into consideration historic earthquakes that occurred in these zones.

At present, there are three major non-canonical collision zones: Alaska, New Zealand, and Indonesia (geophysicists believe that there are probably more, but these three are well known). Since the relative motion of tectonic plates changes over geological time, it could also happen that in the geological past, collisions were non-canonical in some other zones. It is therefore necessary to search for such past zones in different parts of the globe.

In order to understand how much effort we need to invest in such a search, we must first check whether such zones existed at all. At present, there are, basically, three non-canonical collision zones out of several dozens known major collision zones; if we take into consideration minor collision zones, the percentage of non-canonical zones stays approximately the same. Maybe, at some point in the distant past, there were none? Maybe, in the past, all collisions were heads-on, and our search is futile?

In other words, *is it possible that all plate collisions are heads-on?*

In this paper, we will reformulate this question in geometric terms, and show how to solve it.

2. REFORMULATION OF OUR PROBLEM IN SIMPLIFIED TOPOLOGICAL TERMS, AND SOLUTION OF THE CORRESPONDING TOPOLOGICAL PROBLEM

Let us reformulate our geophysical question (is it possible that all collisions are heads-on?) in topological terms.

At any given moment of time, the motion of different points on the Earth surface can be described by assigning to each point x a vector $\vec{v}(x)$ describing the velocity in the direction of the motion. Inside the plate, the directions in which different points move either coincide, or are at least close for nearby points. In other words, on each plate, the dependency of $\vec{v}(x)$ on x is *continuous.* Interaction points correspond to *discontinuities* of the corresponding vector field $\vec{v}(x)$.

If all interactions are canonical (heads-on collisions or pull apart motions), then the only possible discontinuity is going from $\vec{v}$ to a vector that is collinear with $\vec{v}$, but may go in a different direction (i.e., to a vector $\lambda\vec{v}$, where the real number λ can be negative).

We can slightly simplify this picture if, instead of the actual motion vectors $\vec{v}$, we consider *unit* vectors $\vec{d} = \vec{v}/\|\vec{v}\|$ that describe the *direction* of the motion (we assume that no points are immobile, so $\vec{v}(x) \neq 0$ for all x). In terms of the resulting vector field $\vec{d}$, the only possible discontinuity is going from a vector $\vec{d}$ to the opposite vector $-\vec{d}$. In this case, although the vector field $\vec{v}(x)$ is, in general, *dis*continuous, but the following related mapping is *continuous*: the mapping $l(x)$ of every point x from the spherical surface S (of the Earth) to a *line* $l(x)$ (in the tangent space) defined by $\vec{d}$. In other words, a discontinuous vector field $\vec{d}(x)$ defines a *continuous line element field* $l(x)$ (for definitions and properties of line elements fields, see, e.g., (Prasolov 1995), Chapter 6, pp. 56, 61, and 62).

So, the above geophysical question can be reformulated as a following topological problem: *Does there exist a continuous line element field on a sphere?* Our main result is as follows:

Definition. *A linear element field on a smooth manifold V is a mapping that maps every point $x \in V$ into a straight line in the tangent space at x.*

Comment. The notion of continuity can be naturally defined for linear element fields.

PROPOSITION. *No continuous line element fields are possible on a 2-D sphere.*

Comment. In geophysical terms, this result shows that it is impossible for all plates to have only canonical interactions (heads-on collisions or pull apart motions), and therefore, that at any moment of geophysical time, there was at least one non-canonical interaction zone.

Idea of the proof. We will prove the Proposition by reduction to a contradiction. Let us assume that $l(x)$ is a continuous line element field on a 2-D sphere.

The 2-D sphere S is a *simply connected space* in the sense that its *fundamental group* (*first homotopy group*) $\pi_1(S)$ is degenerate ($\pi_1(S) = \{1\}$). For exact definitions, see, e.g., Chapter VIII from (McCarthy 1988); crudely speaking, this means that every continuous closed path on a sphere can be continuously transformed into a point (degenerate closed path).

On spaces with this property, from every continuous line element field $l(x)$, we can construct a continuous vector field $\vec{c}(x)$ such that for every x, the vector $\vec{c}(x) \neq 0$ defines the direction of the line $l(x)$. This construction is given in (Prasolov 1995), pp. 61 and 62: we pick an arbitrary point x_0, choose one of the two possible directions defined by $l(x_0)$ as $\vec{c}(x_0)$, and then for every other point x, choose a curve from x_0 and x and, along this curve, choose one of the two unit vectors at each of the intermediate points so that these vectors vary continuously. The simple connectedness property guarantees that the resulting direction $\vec{c}(x)$ is independent on the choice of a curve going from x_0 to x.

Now, we have a continuous non-zero vector field $\vec{c}(x)$ on a sphere, and this is well known to be impossible; see, e.g., (Shashkin 1991), Chapter 13, p. 60.[1] This contradiction show that our initial assumption (that continuous line element fields are possible) is inconsistent. Thus, there are no continuous line element fields. Q.E.D.

[1] This result is known as the Theorem on a Hedgehog: a hedgehog rolled into a ball has at least one needle that is “sticking out” of that ball.

3. GEOMETRIC APPROACH: AN OPEN PROBLEM

In the above topological formalization, we formalized "canonical interactions" as plate interactions in which the directions of the motion vectors on the two sides of the collision zone are either identical, or exactly opposite to each other. From the geophysical viewpoint, there are two reasons why this is an oversimplicification:

- First, we defined heads-on collision as a one in which the angle between the motion vectors of two colliding plates is *exactly* 180°. In reality, if this angle is 179° or even 170°, this is, for all geophysical purposes, a heads-on collision.
- Second, from the geophysical viewpoint, what matters most is not so much the *absolute* motion of the two plates, but their *relative* motion. If the *relative* velocity of the two plates is orthogonal (or almost orthogonal) to their common boundary, then the corresponding interaction is *almost* canonical.

So, it is natural to ask the following question:

- in this paper, we have shown that it is impossible for every plate interaction to be canonical;
- *is it possible for every plate interaction to be "almost canonical"* (in the above sense)?

This question can be reformulated in purely geometric terms: *Is it possible to have the following geometric structure on a sphere:*

- The surface of the sphere is divided into several *polygons* (i.e., areas bounded by arcs of large circles); these polygons represent different tectonic plates.
- Each polygon is moving as a solid body, i.e., a vector $\vec{v}(x)$ is assigned to each point x, so that when each point slides along this vector, the distance between two points from the same polygon polygon stay the same.
- On every edge (border segment between two polygons), the *relative motion*, i.e., the difference between the two vectors that describe motion of two border polygons, is non-zero and "almost orthogonal" to the border in the sense that the angle α between the relative motion vector and the boundary is *close* to 90° (i.e., $|\alpha - 90| \le \Delta$ for some small $\Delta > 0$).

Is such a configuration possible for all Δ? If not, what is the smallest Δ for which such a configuration is possible?

CONCLUSION

Our result is a somewhat unexpected application of abstract mathematics (topology) to a real-life (geophysical) problem. Even though our current result may not answer the most urgent problems of geophysics, it is an indication that further topological and especially geombinatoric analysis can lead to more serious applications.

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